A hybrid model for customer portfolio analysis in retailing

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Abstract

Purpose – Marketing/finance interface and application of its new insights in marketing decisions have recently found great interest among marketing researchers and practitioners. There is a relatively large body of marketing literature about incorporating modern portfolio theory (MPT) into customer portfolio context and taking advantage of it in marketing resource allocation decisions. Previous studies have modelled customer portfolio risk in the form of historical return/profitability volatility of customer base. However, the risk is a future-oriented measure, and deals with future volatility associated with return stream. This study aims to address this research problem.

Design/methodology/approach – The well-known Pareto/non-binomial distribution (NBD) approach is used to model customer purchases in a non-contractual setting of research practice. Then, the results were used to simulate the customers' future buying behaviour and associated returns via the Monte Carlo simulation approach. Subsequently, the mean-variance portfolio optimization model was applied to find the optimal customer portfolio mix.

Findings – The results illustrated the better performance of the proposed efficient portfolio versus the current customer portfolio. These results are applicable in analyzing customer portfolio composition, and can be used as a guidance to make decisions about marketing resource allocation in different segments.

Originality/value – This study proposes a new approach to analyze customer portfolio by using the customers' future buying behaviour. Taking advantage of rich marketing literature about statistical assumptions describing the customers' buying behaviour, this study tries to take some steps forward in the application of the MPT theory in customer portfolio management context.

Keywords Monte Carlo simulation, Pareto/NBD model, Customer portfolio management (CPM), Marketing/finance interface, Return-on-marketing

Paper type Research paper

1. Introduction

Managing the customer base in a way that maximizes the profit performance and stabilizes cash flow is of great interest to marketing managers. One promising line in this regard is customer portfolio management (CPM). As discussed by Srivastava *et al.* (1998), customers as intangible assets could be conceptualized as financial assets. The concept of CPM is taken from financial portfolio theory, which states that overall performance of assets in a portfolio should be measured by considering the trade-off between their risk and returns. The modern portfolio theory (MPT) attempts to maximize portfolios' expected returns for a given level of portfolio risk (Markowitz, 1952). The idea behind this theory is based on the concept of diversification in investment: selecting collection of assets, which collectively have lower risk than selecting them individually. The idea of diversification has also been accepted and used in the marketing literature. An example of it is reducing the dependency on one large

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customer, and therefore, the risk of his/her defection by getting involved with other customers. However, MPT goes beyond simple diversification and tries to compose a portfolio of assets with low correlation of return streams and thus the minimum overall risk (Selnez, 2011). One question that might be raised is how to define the customer risk? In financial applications, the uncertainty associated with cash flow is defined by the difference between the expected returns and the actual amounts, which are realized. This uncertainty is characterized as "risk". The variability for an asset is commonly characterized by deviation of its returns from their expected value during the holding period and is commonly calculated by the variance of cash flows. Hence, the earliest and the most common measure for evaluating risk in the marketing literature is the variance of cash flow generated by customers. Unfortunately, most of the studies in this regard have used the volatility of historical sales or profit made by customers as a proxy of customer risk (Ryals, 2003; Buhl and Heinrich, 2008; Juhl and Christensen, 2013; Kundisch *et al.*, 2008; Tarasi *et al.*, 2011). To the best of our knowledge, there is no report in the literature that has tried to make an ex ante measurement of customer risk.

The present study attempts to fill this gap and estimate the volatility of future returns generated by customers. In traditional financial applications, it is assumed that returns generated by underlying securities follow a normal distribution function. In this study, we incorporate some statistical assumptions describing customer purchase behaviour into customer portfolio analysis. Due to the non-contractual setting of our research practice, we use Pareto/non-binomial distribution (NBD) model to extract these statistics. The results are used in a Monte Carlo (MC) simulation procedure to produce the expected customer purchase pattern over time and to subsequently build the customer portfolio. Section 3 deals with the proposed approach in detail.

The paper is organized as follows: Section 2 gives a review on the research background about CPM from different aspects. Section 3 introduces the research design and the phases of empirical research. It also provides some brief descriptions about the modelling techniques used in this study. The proposed model is introduced and applied through five phases in Section 4. In Phase 1, we used the Pareto/NBD model to estimate statistics about the customers' buying behaviour. In Phase 2, based on the results gained from the previous phase, we used the MC simulation approach to draw customer buying behaviour in the holdout period. In Phase 3, according to the cost of goods sold and the monthly variable costs associated with the customer segments, we derived the monthly return on sale for every customer segment. In Phase 4, we calculated the possible weights considered for the customer portfolio according to the managers' viewpoints. Finally in Phase 5, the return streams provided from Phase 3 were combined to form a customer portfolio. We used mean-variance (M-V) optimization approach to identify its efficient frontier. The empirical phases of this study and the models used in every step are illustrated in Figure 1.

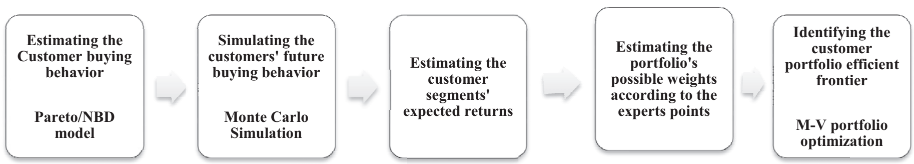


Figure 1.
Empirical phases of research practice

2. Research background

Early customer portfolio models (similar to those of Fiocca, 1982; Campbell and Cunningham, 1983; Dubinsky and Ingram, 1984; Yorke and Droussiotis, 1994) focused on customers' profitability and strategic management of their relational portfolio in a way to ensure their long-term profitable relationships. In these models, the aspect of risk was widely neglected. However, considering both aspects of providing higher profitability and lowering variability of customers' cash flow is central for marketing managers. Tuli *et al.*'s (2010) results imply that extending the number and types of relationships ties with high-value customers ensures higher profitability and lower variability of customers' purchases. Bowman and Narayandas (2004) also provide evidence that identifying items in customer management efforts that provide higher marginal returns on additional investment, can lead to higher returns. The aspect of risk is usually incorporated in customer lifetime value (CLV) models by means of risk-adjusted discount rate (Hogan *et al.*, 2002; Hopkinson and Lum, 2002; Ryals, 2002, 2003). In this approach, a customer value with less stability of cash flow is evaluated less than the customer with more stable cash flow.

MPT provides a straightforward approach to consider both aspects of profitability and variability in portfolio management. Cardozo and Smith's (1983) research is the first report on the application of MPT in a non-financial application. They developed this theory to make product portfolio decisions. Although their results were criticized by Devinney *et al.* (1985) because of what they identified as key differences between financial assets and product investment assets, this study generated a research stream to incorporate MPT on other disciplines. Also Archer and Ghasemzadeh (1999) developed the application of MPT into project portfolio management.

There are some other studies about CPM from the aspect of incorporating MPT into customer portfolio analysis. The main advantage of applying the MPT approach into customer portfolio is in considering interdependencies between the assets' risk structure. Johnson and Selnes (2005) expressed this concept as "analyzing the forest, rather than the trees". Dhar and Glazer (2003) made a noteworthy contribution in this field. They introduced a measure called "Customer Beta", which measured the riskiness of a customer compared with the whole portfolio. In their approach, the firm could evaluate the risk associated with different customers/customer segments according to the contribution of their cash flow patterns on the overall customer base cash flow. Sackmann *et al.* (2010) tried to investigate the efficient frontier using CLV of customers within a segment instead of cash flow generated by that segment. Then M-V optimization problem was used to find the frontier with maximum expected portfolio's CLV while maintaining a certain level of CLV volatility in the segments. Buhl and Heinrich (2008) enhanced the application of MPT in CPM and provided a framework to examine how adding or subtracting different customer segments could affect on the whole portfolio. There is also a research stream which deals with CPM in terms of dynamics in customer segments. Johnson and Selnes' (2004) study is the first to investigate the customer portfolio dynamics. Using a set of propositions about customer segment dynamics, they conceptually extract foundations for CPM. Johnson and Selnes' theoretical framework has been applied in practice by Homburg *et al.* (2009). Terho and Halinen's (2007) study is another study in this regard to display how a specific firm conducts dynamic portfolio analysis through multiple case studies. Terho (2009) extended this idea and developed a new formative measure to study how customer portfolio analysis efforts affect firms' performance. Abascal *et al.*'s (2010) and Wangenheim and Lentz's (2005) research are one of few that studied the changes in customer portfolio composition over time. Tarasi *et al.* (2011) made a significant contribution in this regard by explicitly incorporating the MPT in CPM in a business-to-business research practice. They studied variability of revenues generated by segments as risk. Selnez (2011) criticized this approach and argued that the revenue stream may not necessarily be consistent with returns. Tarasi *et al.* (2013) also studied variability in customer relationships as variability in service consumption patterns. In their study, they showed that variability in customers' cash flow can be linked to underlying variability in the service consumption process.

However, there are some criticisms about incorporating MPT in case of customer assets. Selnez (2011) and Billet (2011) made some comments about Tarasi *et al.*'s (2011) work from various managerial and practical issues points of view. These comments mainly relate to differences between customer assets and financial assets that have been discussed in the following sections. It is important to note that MPT makes many assumptions about assets; some of which may not be met well in the case of customer portfolio assets. Therefore, as Devinney *et al.* (1985) commented on Cardozo and Smith's (1983) work, the financial portfolio theory needs some modifications to be incorporated into the customer portfolio context.

One of the major assumptions in MPT is to assume a complete and liquid market for trading assets. In such a market, assets are arbitrarily divisible and an investor can buy or sell any desired amount of securities whenever he/she wants. It is obvious that in the customer portfolio context, in contrast to financial markets, there is no liquid market for trading securities and the marketing manager could only affect the portfolio composition by changing the resources assigned to different market segments (Reinartz *et al.*, 2005; Venkatesan and Kumar, 2004). In other words, the finite supply of customers of different segments limits the feasible area of weighting schemes, which could be realized (Billet, 2011). Due to this practical constraint, it seems that identifying the feasible portfolio weights through a qualitative approach in accordance with the case study could resolve this problem.

One other major concern in the application of MPT in finance literature is the prediction of mean and covariance of the assets' return. In traditional MPT, it is assumed that returns follow a normally distribution pattern. Despite the fact that this assumption is violated in some special cases of financial assets, its assumption has a strong theoretical foundation in the financial context (Fama, 1965). Nevertheless, translating this idea into the customer portfolio context requires considering customers' purchasing behaviour and predicting their expected purchase patterns in the future. Selnez (2011) argued that incorporating customer purchase patterns into customer portfolio theory could advance this stream greatly.

Another aspect relates to the nature of risk in customer relationships. Before quantifying the risk in customer relationships, its sources must be identified (Hogan *et al.*, 2002). The customer risk is mainly because of their defection (Wangenheim and Lentz, 2005). Therefore, finding an appropriate way to consider this source of risk is necessary for developing any model in this regard. In non-contractual setting, this is somewhat difficult because the defection process is not observable. A stochastic model of purchase pattern is an appropriate choice for considering customer defection. In such a model, the probability of defection at any time period is calculated as a probability function of a customer's past behaviour. The Pareto/NBD is a powerful stochastic model in a non-contractual setting.

In the present research, taking advantage of the issues argued in this section, we tried to take some steps forward and consider some restrictions about incorporating MPT into the CPM context. However, in addition to the aspects argued in this section, there exist other criticisms about incorporating MPT in the customer portfolio context which we will discuss in the conclusion section.

Here, for the first time, we will examine the expected future returns generated by customers instead of their historical values. Risk and return associated with securities in a financial portfolio are future-oriented quantities. Thus, our approach makes more tangible insights about the performance of portfolio in future. However, to "translate" these concepts into the customer portfolio context, some specific characteristics of customer portfolio should be considered. Here, we used the Pareto/NBD model to predict customers' purchase behaviour.

In addition, we introduce an approach to take into account the customer purchasing behaviour in CPM and construct a portfolio with the right composition of customer purchase patterns. In addition, to consider a feasible portfolio-weighting scheme in our model, we use managerial judgement about feasible weighting area and apply it in terms of constraints in a mathematical model.

3. Research design

3.1 Study context

The case study of this research included a distributer and online retailer of cosmetic products in Tehran. This firm sells its products via two channels. The first is related to online retailing, and the latter is via marketers. The customers of e-tailing are usually consumers from all over Iran, while the customers related to marketers are usually from small stores and beauty salons in Tehran. Hereinafter, we call the first segment as "internet customers", and the latter as "marketing customers". The data provided for this study included two years of data about the customer' purchases (from 21/03/2010 to 20/03/2012). We divided this time duration into two intervals. The first 12 months were considered as the calibration period and the next 12 months as the holdout period. We focus on the customers who had at least one transaction in the calibration period. During the first year, 1,411 customers consisting of 1,305 internet customers and 106 marketing customers had relationships with the firm. During the calibration period, these customers made 2,095 and 199 transactions with the firm, respectively. In a similar way, they also made 1,367 and 216 transactions in the holdout period. The data needed for this study were gathered from various sources of the firm. We derived the customers' buying behaviour from the transactional dataset (for the internet customers) and from the customer invoices (for the marketing customers). These data fields have been used to form a dataset containing customer ID, date of transaction and monetary value. The monthly cost of goods sold has been derived from the documents of storage room. The other needed data about the monthly variable cost have been extracted from the marketing department. The detailed descriptions about these variables' costs for two segments of customers are provided in Section 4.

3.2 Pareto/NBD model

Pareto/NBD, which was first introduced by Schmittlein *et al.* (1987), is a probability approach to model CLV. Probability models assume that customers' behaviour varies across the population according to a specific distribution. Pareto/NBD model is a powerful tool to describe customers' behaviour in a non-contractual setting (Fader *et al.*, 2010; Mulhern, 1999; Niraj *et al.*, 2001; Reinartz and Kumar, 2003; Schmittlein and Peterson, 1994); therefore, we used it to estimate the customers' purchase attributes in this research. This model makes some assumptions about customers' buying behaviour[1] (Fader *et al.*, 2005):

- Customer relationships with a firm consist of two phases; in Phase 1, he/she is active across an "unobserved" time period, and in Phase 2, he/she becomes inactive permanently.
- When the customer is still active, the number of transactions follows a Poisson distribution.
- Heterogeneity in the number of transactions follows a gamma distribution across the population.
- Customer's lifetime duration has exponential distribution.
- Heterogeneity in the customers' dropout rates follows a gamma distribution across the population.
- Transaction and dropout rates vary independently from each other.

The second and third assumptions above yield exponential distribution for inter-purchase times ($IPT_{ij} \sim Exponential(\lambda_i)$); where, its parameter itself follows a gamma distribution among the customers ($\lambda_{ij} \sim Gamm(r,\alpha)$). The fourth and fifth assumptions result in the Pareto of the second kind distribution for customer lifetime duration ($totLife_i \sim Pareto(s,\beta)$).

As specified above, r and α are the parameters of a probability distribution which estimates the transaction rate, and s and β are the parameters of a distribution which estimates the dropout rate. These four parameters are estimated using the likelihood function defined by equation (1). This model uses three pieces of information about the customers' past behaviour: recency (when the last transaction occurred), frequency (how many transactions have occurred) and the time period within which the customer's behaviour has been observed. The notation (x_i, t_i, T_i) represents these parameters, where, x_i is the number of transactions made by customer i in the time period $(0, T_i]$ and t_i $(0 < t_i \leq T_i)$ is the time of customer's last transaction. These assumptions lead to Pareto/NBD likelihood function. This function for customer i with the purchase history of (x_i, t_i, T_i) is denoted as $L_i = L(r, \alpha, s, \beta | x_i, t_i, T_i)$ and can be calculated as:

$$L_i(r, \alpha, s, \beta | x_i, t_i, T_i) = \frac{\Gamma(r + x_i)\alpha^r \beta^s}{\Gamma(r)} \left\{ \frac{1}{(\alpha + T_i)^{r+x_i}(\beta + T_i)^s} + \left(\frac{s}{r + s + x_i} \right) A_0 \right\} \tag{1}$$

In the above equation, if $\alpha \geq \beta$, then A_0 is:

$$A_0 = \frac{{}_2F_1\left(r + x_i + s, s + 1; r + x_i + s + 1; \frac{\alpha - \beta}{\alpha + t_i}\right)}{(\alpha + t_i)^{r+s+x_i}} - \frac{{}_2F_1\left(r + x_i + s, s + 1; r + x_i + s + 1; \frac{\alpha - \beta}{\alpha + T_i}\right)}{(\alpha + T_i)^{r+s+x_i}} \quad (2)$$

Otherwise, if $\alpha < \beta$, then A_0 is:

$$A_0 = \frac{{}_2F_1\left(r + x_i + s, r + x_i; r + x_i + s + 1; \frac{\beta - \alpha}{\beta + t_i}\right)}{(\beta + t_i)^{r+s+x_i}} - \frac{{}_2F_1\left(r + x_i + s, r + x_i; r + x_i + s + 1; \frac{\beta - \alpha}{\beta + T_i}\right)}{(\beta + T_i)^{r+s+x_i}} \quad (3)$$

Some extensions from this basic model have been developed. One is a transactional value model, which incorporates the customer transactions' monetary values into the model. Fader *et al.* (2005) made some assumptions in this regard:

- The monetary values generated by a customer vary independently from his/her number of transactions.
- The monetary values follows a gamma distribution.
- Heterogeneity in monetary values has a gamma distribution across the customers.
- Mean monetary values differ among the customers but do not vary over time for any given customer.

The second assumption states that monetary values (m_i) have a gamma distribution as ($m_i \sim \text{Gamma}(px_i, v_i)$). The third assumption indicates that the monetary values' scale parameter itself follows a gamma distribution among the customers ($v_i \sim \text{Gamma}(q, 1/\gamma)$). These assumptions lead to transactional likelihood function as follows:

$$L(p, q, \gamma) = \prod_{i=1}^{n} \left(\frac{\Gamma(px_i + q)}{\Gamma(px_i)\Gamma(q)} \frac{\gamma^q \bar{m}_i^{px_i - 1} x_i^{px_i}}{(\gamma + \bar{m}_i x_i)^{px_i + q}} \right) \quad (4)$$

The parameters of likelihood functions in equations (1) and (4) could be estimated via the maximum likelihood estimation (MLE) approach.

3.3 Mean-variance optimization model

As discussed earlier, the basis of MPT is about reducing the risk through investment diversification. The M-V optimization model is a straightforward approach to study the performance of a portfolio in terms of trade-off between reward and risk. In this model, the reward is measured by the portfolios' mean return and the risk by its variance.

Markowitz (1952) mathematically proved that the portfolio volatility is a function of the correlation of assets comprising it. The trade-off between the portfolio's overall risk and return can be illustrated in a two-dimensional diagram. The investor can change his/her portfolio value by changing the proportions of various assets in the portfolio combination. There are some fundamental assumptions in the M-V model about selecting the portfolio. The portfolio should be feasible. It means that the portfolio weights should vary in [0, 1] interval and all portfolio weights should sum up to one. The other assumption is that if any portfolio has greater variance than any other portfolio, it should yield higher return to provide investment attractiveness and vice versa. In this model, only the first two moments (mean and variance) are considered in the portfolio model; therefore, it is called the M-V model.

In the mathematical formulation of this model, consider n assets comprising a portfolio. We denote w_i as the percentage of wealth invested in asset i, r_i as the return of asset i, μ_i as its expected return, σ_i as its standard deviation and $\sigma_{i,j}$ as the covariance value of returns of assets i and j, where $i, j = 1, \ldots, n$. We also denote ρ as a given expected return of whole portfolio.

Following the Markowitz approach, we have to determine the set of portfolio weights in a way that minimize the variance for a given expected return. This leads to the following quadratic optimization problem which is known as the classical M-V model:

$$Min\ \sigma_P^2 = Min \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j Cov(r_i, r_j) = Min \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} w_i w_j \tag{5}$$

st:

$$\begin{gathered} \sum_{i=1}^{n} \mu_i w_i = \rho \\ \sum_{i=1}^{n} w_i = 1 \\ w_i \geq 0 \quad i = 1, \ldots, n \end{gathered} \tag{6}$$

The solution of this optimization model is called the "efficient frontier", which is a set of points that has higher returns than any other possible portfolio with the same level of risk or have the minimum risk compared to other possible portfolios with the same level of expected return.

In a general form, the efficient frontier is a set of points that defines an arc of hyperbola in risk/return diagram. In other words, the efficient frontier is the upper portion of the minimum variance curve. This curve is illustrated in Figure 2.

All points on the efficient frontier curve have the same target expected return but lower variance than points on the right of the curve. In other words, all points that lie under this curve are possible and sub-optimal portfolios, and points that lie above it are infeasible ones. In this case, the feasible portfolios are placed inside a convex curve.

Different points of this curve represent different levels of required expected return. As long as reducing the expected return leads to reduced variance, a reasonable investor may be satisfied. But if the expected return is as low as that variance has to increase to reach it, he would not (Zenios, 2007). So the portfolio with the lowest possible variance is of particular interest for investors.

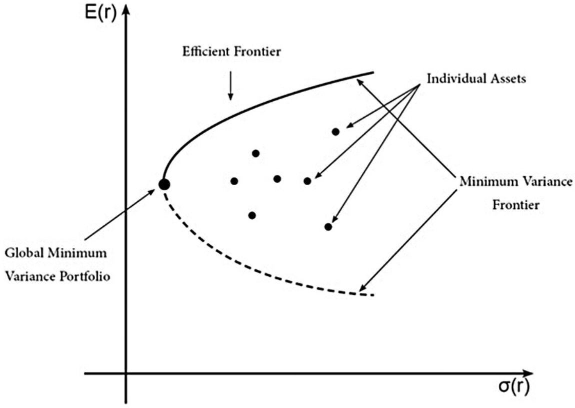


Figure 2.
Minimum variance and efficient frontiers for portfolio consisting from more than two assets

In a case of two asset portfolios, where we have a percentage of wealth (w_1) invested on the first asset and ($1 - w_1$) invested on the second asset, the above problem is converted to a univariate optimization problem. Assume that A and B are these two assets. As illustrated in Figure 3, we can represent these points in an M-V diagram. A set of all possible combinations of the two assets can be displayed by a curve connecting these two points. The shape of this curve depends on the covariance between the two assets' returns. Figure 3 displays this curve for imperfectly correlated risky assets ($-1 < \rho$A,B $< +1$). For perfectly correlated assets, this curve would be a straight line. As illustrated in Figure 3, with only two risky assets, there are at most two different levels of expected return (E) for a given level of risk (σ), but with three or more risky assets, there can be many levels of E for a given σ.

It should be noticed that only the upper arc of this curve is efficient. Other points below the minimum variance point (which we illustrated them by dashed line) are feasible but inefficient portfolios.

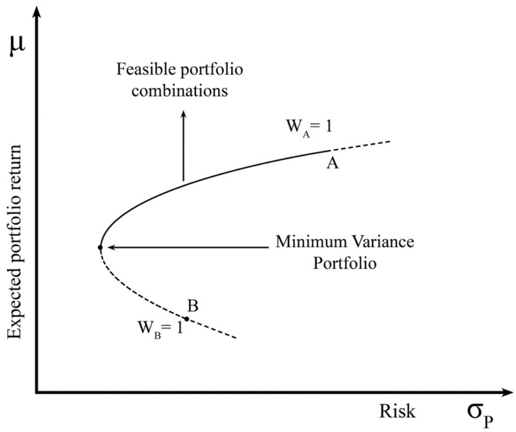


Figure 3.
Portfolio frontier for two risky assets, A and B

In this case, the return of portfolio P, which combines the two risky assets A and B, is:

$$r_p = w_A . r_A + \mathrm{w_B} . r_B \tag{7}$$

$$\text{Where, } \mathrm{w_A} + \mathrm{w_B} = 1 \text{ and } 0 < w_A < 1 \tag{8}$$

The variance of the portfolio as a measure of the portfolio's risk is:

$$\sigma_P^2 = w_A^2 . \sigma_A^2 + w_B^2 . \sigma_B^2 + 2 w_A \mathrm{w_B} \sigma_A \sigma_B \rho_{AB} \tag{9}$$

Where, the term $\sigma_A \sigma_B \rho_{AB}$ is equivalent to cov $\{r_A, r_B\}$.

Combining the equations (7)-(9) represents a hyperbola in the mean-standard deviation space, which illustrates the feasible combinations of mean and standard deviation. This curve is illustrated in Figure 3.

To achieve the weights of minimum-variance portfolio, we should derive the weight of security A that minimizes the portfolio's standard deviation σ_P. By computing the term $d\sigma_P/dw_A = 0$, the result would be as below:

$$\mathrm{w_A} = \frac{\sigma_B^2 - \rho_{A,B}\sigma_A\sigma_B}{\sigma_A^2 + \sigma_B^2 - 2\rho_{A,B}\sigma_A\sigma_B} = \frac{\sigma_B^2 - \mathrm{Cov}(r_A, r_B)}{\sigma_A^2 + \sigma_B^2 - 2\mathrm{Cov}(r_A, r_B)} \tag{10}$$

This value of w_A represents the minimum variance portfolio, which has been illustrated in Figure 3. In this figure, as w_A varies from 1 to 0, a convex curve is formed from points A to B.

4. Experimental results

4.1 Phase 1: Estimating the customers' buying behaviour via the Pareto/NBD model

As mentioned in the previous section, we used the Pareto/NBD model to extract some attributes about the customers' purchase behaviour. The parameters of this model and its extension about the monetary values are estimated by the MLE approach. We used our data set containing the customer ID, date of transaction and monetary value to build a secondary data set. The fields existing in the second dataset are:

- total number of transactions in the calibration period (x_i);
- time interval between the customer's first and last transactions in the calibration period (t_i);
- time interval between the customer's first transactions until 20/03/2011 (the end of calibration period) (T_i);
- the average monetary value of transactions made in the calibration period (m_i);
- number of transactions made in the holdout period ($\acute{x}_i$);
- the average monetary value of transactions made in the holdout period ($\acute{m}_i$); and
- the time interval between the customer's first transaction until his/her last transaction in the holdout period (if exists) ($\acute{t}_i$).

The time unit we used in these calculations is month. The monetary values have been reported in Rial 10,000 for ease of calculations. We applied the first four fields of this data set to construct the Pareto/NBD model (Schmittlein *et al.*, 1987) and its extension of a transactional model (Fader *et al.*, 2005). Then these data fields are denoted as $X_i = \{x_i, t_i, T_i, m_i\}$. For example, $X_{100} = \{2, 4, 5, 35\}$ indicates that the 100th customer had two transactions in the calibration period, and the time lapse between his/her first transaction till the end of the calibration period was five months. In other words, he/she had made his/her first transaction in Month 7 and his/her last transaction four months later in Month 11. The average value of these two transactions was Rial 350,000.

We extracted these data fields for all 1,411 customers and calculated the likelihood functions defined by equations (1) and (4) for all customers. The likelihood function stated by equation (1) is a function of r, α, s and β parameters, while the latter is a function of p, q and γ parameters. Then we calculated the sum of log-likelihood values and estimated its minimum value using *fmincon* function in the Matlab optimization toolbox. These calculations are carried out for each customer segment separately. The results are shown in Table I.

4.2 Phase 2: Simulating the customers' future buying behaviour using Monte Carlo simulation

We used a simulation-based approach to estimate the customers' future purchase behaviour. As mentioned in Section 3, we made some distributional assumptions about some statistics describing the underlying customers' behaviour according to the Pareto/NBD model. In this model, customer defection is modelled by death process. Table II summarizes these assumptions in the above model.

However, according to these statistical distributions, we drew random numbers about the lifetime duration, inter-purchase times and monetary values for two customer segments. The actual values of these parameters during the calibration period are shown in Table III.

Taking advantage of the results gained in Phase 1, we executed an MC simulation procedure to estimate the customers' expected buying behaviour in various time periods.

MC simulation is a computational approach that relies on simulating the underlying process through a repeated random sampling, and obtains the desired average results of

Table I.
Parameters of Pareto/NBD and transactional value models

Customer segment	r	α	s	β	p	q	γ
Marketing customers	0.4833	1.9653	1.1942	12.9760	3.7291	11.1406	3,568.5318
Internet customers	0.9552	7.8271	1.2735	3.0425	1.2509	2.0821	25.1294

Table II.
Distributional assumptions about the customers' buying behaviour

Statistic	Distributional assumption	Distribution formula
Customer's lifetime duration	$Lifetime_i \sim Pareto(s,\beta)$	$f(Lifetime_i \mid s,\beta) = \frac{s}{\beta}\left[\frac{\beta}{\beta+Lifetime_i}\right]^{s+1}$
Customer's inter-purchase times	$IPT_{ij} \sim Exponential(\lambda_i)$	$f(IPT_{i,j} \mid \lambda_i) = \lambda_i e^{-\lambda_i IPT_{i,j}}$
Customer transactions' monetary values	$m_{ij} \sim gamma(px_i,v_i)$	$f(m_{i,j} \mid p,x_i,v_i) = \frac{(v_i x_i)^{px_i} m_{i,j}^{(px_i-1)} e^{(-v_i x_i m_{i,j})}}{\Gamma(px_i)}$

that process. This experiment can be executed by drawing lots of random samples and observing their behaviour. This procedure is basically as follows (Mooney, 1997):

- Specify a pseudo population in such a way that it can be used to generate samples.
- Draw samples from the pseudo population that reflects the statistical situation of interested results.
- Calculate the interested statistics of the pseudo sample.
- Repeat the above two in numerous trials.

This approach has been applied in various areas such as physics, computer science, finance, etc. (Mooney, 1997).

In financial applications, the MC approach is usually used to model the uncertainty associated with different financial instruments. In these applications, simulation generates several hundred possible random scenarios about the prices and interest rates, which lead to different positions. In portfolio optimization applications, in each trial of simulation, according to the probability distribution of instruments' prices, their correlated behaviour is simulated, and the resulting portfolio value is observed. The notations used in the MC simulation steps are described in Table IV.

The simulation steps are as specified in Table V.

As specified in Table IV, we assume the customers' attraction rate based on the actual rate of new customers' entrance into relationships in both segments over the calibration period (i = 1, …, 12). For the holdout period, we simply assume the same

	Segment 1 (Internet)			Segment 2 (Marketing)		
Statistics	Lifetime duration (in months)	Inter-purchase time (in months)	Monetary values (in Rial 10,000)	Lifetime duration (in months)	Inter-purchase time (in months)	Monetary values (in Rial 10,000)
Mean	5.3667	2.6666	12.4470	6.8000	2.8000	942.0255
SD	6.4667	2.7333	33.5549	6.3333	3.1000	1.0206e + 03
Maximum	23.8000	11.9333	331.7500	21.1666	11.9000	4.5938e + 03
Minimum	0	0.0333	0	0	0.1000	0
Median	3.0333	1.4000	0	6.2000	2.2666	790.5030

Table III. Summary of statistics

Notation	Explanation
i	Customer i in transactional dataset
j	The jth transaction made by a customer during his/her lifetime duration
θ	The θ th time period in calibration period
T_i	The time interval between the customer i's initial purchase time and the current time
t_i	The time interval between the customer i's initial and last transaction times
x_i	Number of transactions made by customer i during time period (0, T_i]
$Lifetime_i$	Lifetime duration of customer i from his/her initial transaction until defection
$m_{i,j}$	The monetary value of jth transaction made by customer i,
$IPT_{i,j}$	The inter-purchase time between the ($j-1$)th and jth transactions of customer i,
Att_θ	Rate of customer attraction in the time period θ
$pt_{i,j}$	Purchase time of customer i in the jth transaction.

Table IV. Presentation of the notations used in this research

Table V.
Simulation steps for the proposed model (using Pareto/NBD assumptions)

Description	Step
Step 0 Initialization	Consider the parameters (r, α, s, β, p, q, γ) from the results of Pareto/NBD and transactional value models for every customer segment Implement the following steps for each of the customer segments
Step 1 Considering customers' entrance in the model	Consider the rate of attraction in time period θ as Att_θ, $\theta = 1, \ldots, 12$ For Att_θ number of customers, assume the first purchase time $pt_{i,1} = \theta$
Step 2 Generating the customers' lifetime duration	Generate random numbers from Pareto distribution of second type with parameters (s,β) to draw $Lifetime_i$ $Lifetime_i \sim Pareto(s,\beta)$
Step 3 Generating inter-purchase times	Generating random numbers from gamma distribution with parameters (r,α) $\lambda_i \sim Gamma(r,\alpha)$ Use this parameter to draw random inter-purchase times $IPT_{i,j} \sim Exponential(\lambda_i)$
Step 4 Determining purchase times	Determine customer i's purchase times according to the customer's random lifetime and inter-purchase times If $(pt_{i,j-1} + IPT_{i,j}) \leq (pt_{i,1} + Lifetime_i)$ assign $pt_{i,j} = pt_{i,j-1} + IPT_{i,j}$
Step 5 Generating the monetary values' stream	Generate random numbers from gamma distribution with shape parameter q and scale parameter γ $v_i \sim Gamma(q,\gamma)$ Use v_i to draw random monetary values $m_{i,j} \sim Gamma(px_i, v_i)$ Where, x_i is the number of customer i's purchases within the calibration period The generated monetary values are assigned to the corresponding $t_{i,j}$.
Step 6	Repeat the above steps for 1,000 rounds of simulation

attraction rate for different months during the holdout period. In this modelling, the dropout rate is determined by lifetime duration parameter. Other characteristics related to the inter-purchase times and the monetary values are determined based on their associated statistical distributions. We executed this simulation procedure for 1,000 trials. The obtained results and their comparison with actual customer buying behaviour are illustrated in Figures 4 and 5.

Using the attributes' estimations provided by Pareto/NBD modelling, the monthly sales associated with every customer segment was simulated (Figure 6).

4.3 Phase 3: estimating the customer segments' future return in the holdout period

Calculations carried out in Phase 2 provide estimations about the lifetime duration, the time of transactions occurrence and their monetary values. These estimations lead to a monthly sale stream for every customer segment. Considering the monthly costs, the monthly return from each segment can be calculated using the following equations:

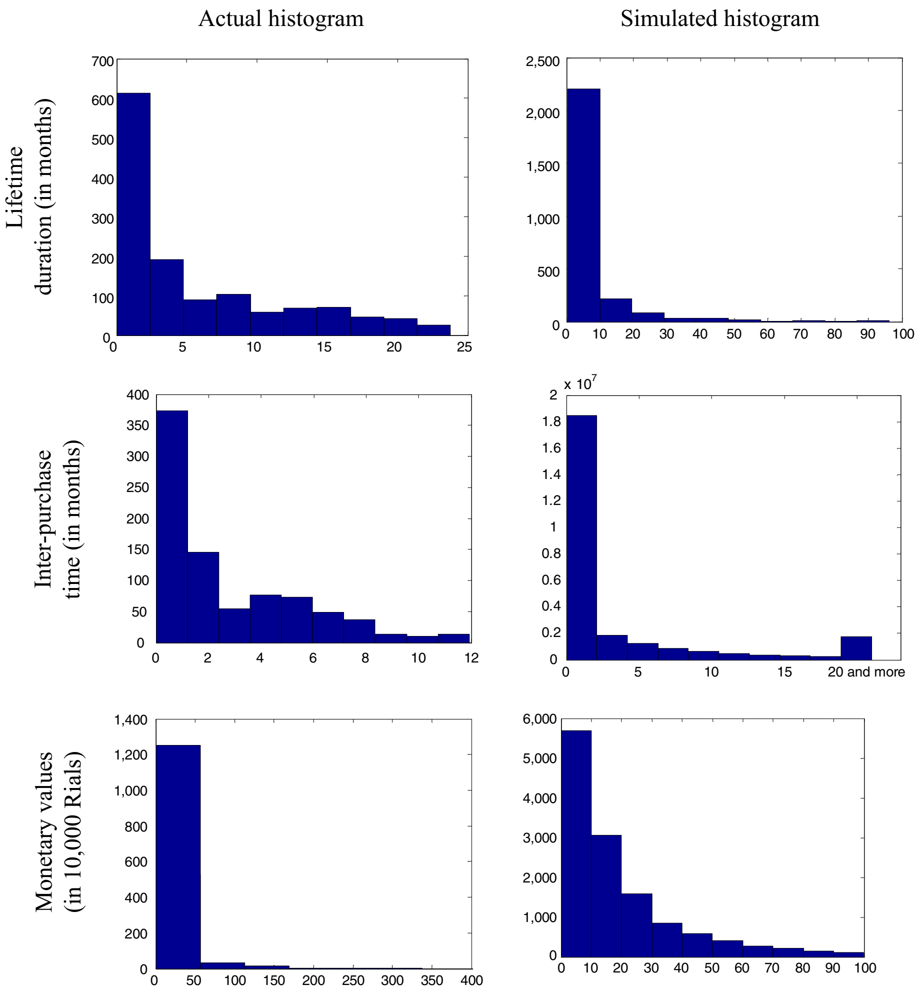


Figure 4. Internet customer features' simulated histograms versus their actual histograms

$$\text{Return on Sales }(\text{ROS}_{i,j}) = \frac{S_{i,j} - C_{i,j} - V_{i,j}}{S_{i,j}} \tag{11}$$

Where, $S_{i,j}$ is the total sale made by segment i during j^{th} time period, $C_{i,j}$ is the cost of goods sold to the customers of segment i during j^{th} time period and $V_{i,j}$ is the total variable cost associated to the customers of segment i during j^{th} time period.

The ROS parameter indicates that how much gross profit we earn from every Rial 1 we sold to different customer segments. Equation (11) only focuses on the variable costs. Since the fixed costs are constant and are not influenced by managerial decisions about marketing resource allocation, they have not been intervened in this equation. We used the simulated monthly values generated by the customer segments as $S_{i,j}$ in equation (11). To calculate the cost of goods sold to each customer

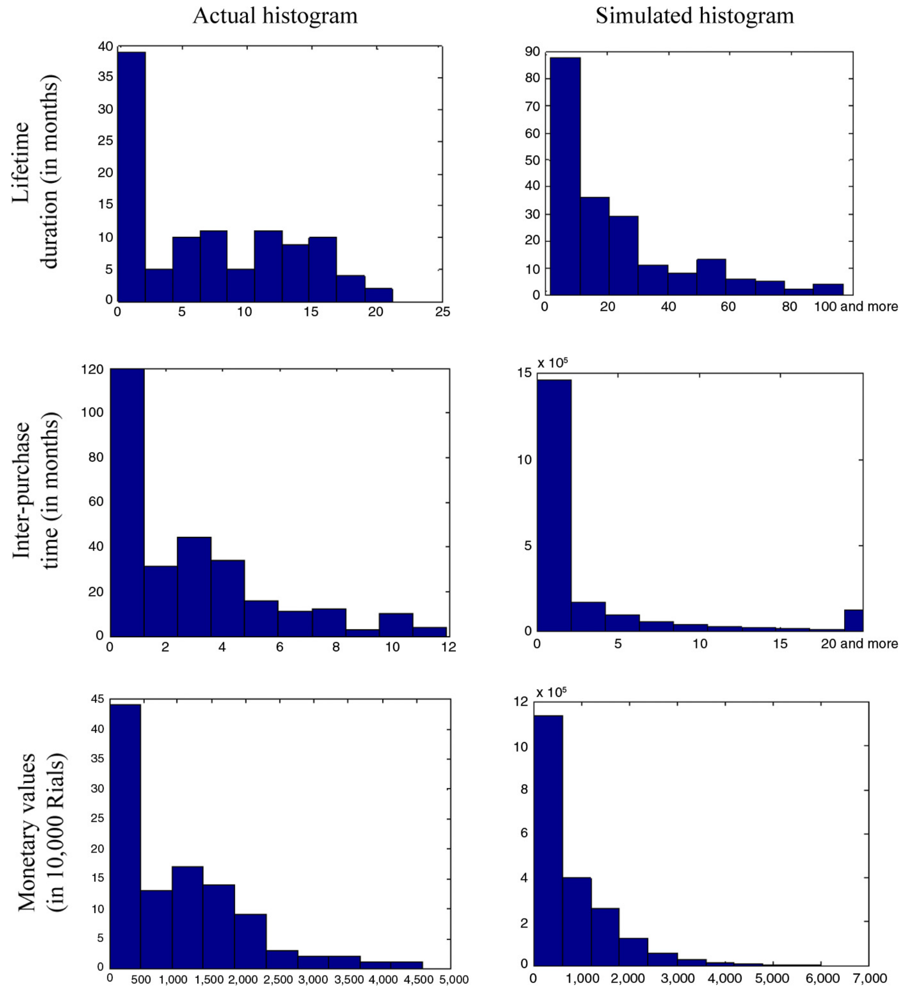
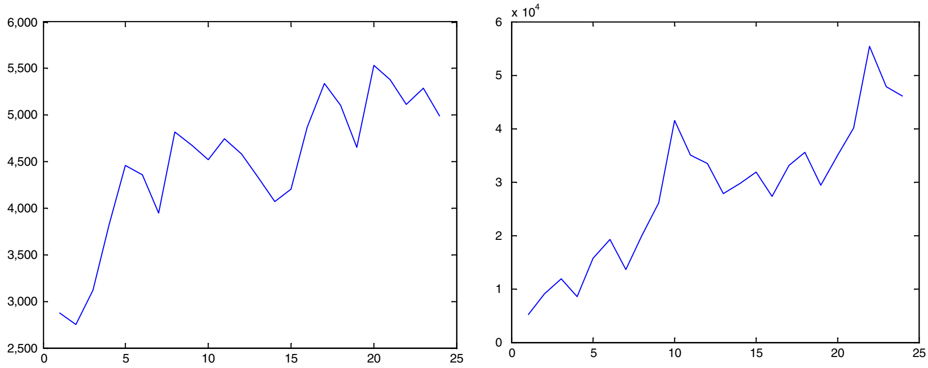


Figure 5.
Marketing customer features' simulated histograms versus their actual histograms

segment, we considered the cost of goods sold in the calibration period. This parameter is divided into monthly sale during the calibration period (i = 1, …, 12). We assumed that these ratios are the same as in the holdout period. Thus, the cost of goods sold in the holdout period is calculated as a product of this ratio multiplied to the monthly sale in the holdout period.

The following section provides detailed descriptions about the variable costs associated with the two customer segments.

(1) *Marketing customers*: For this segment, variable costs include monthly marketers' salary (seven marketers), cost of commissions paid to the marketers (which is calculated as a percentage of monthly total sale made by a marketer from 2 to 5 per cent according to Table VI) and managerial special rewards for the marketers.



Notes: (a) Internet customers' monthly sales; (b) marketing customers' monthly sales

Figure 6. Simulated monthly sales of two customer segments

The cost	The cost item	Descriptions
Variable costs associated with the marketing segment	Monthly marketers' salary	
	Marketers' commission	For sale volumes between Rial 10-20mn, 2% of the total sale For sale volumes between Rial 20-40mn, 3% of the total sale For sale volumes between Rial 40-80mn, 4% of the total sale For sale volumes above Rial 80mn, 5% of the total sale
	Managerial special rewards	
Variable costs associated with the internet segment	Postal charges	5% of the price of goods sold
	Click/banner advertisements	This cost is paid annually and its monthly amount is calculated by dividing this value by 12
	Returned postal costs	

Table VI. Variable costs associated with two customer segments

(2) *Internet customers*: In this segment, variable costs include postal charges associated with the internet sale, which is paid to Iran's Post company; the cost of online click/banner advertisements, which is paid annually to an internet advertising firm; and postal costs associated with the returned postal orders (which occur because delay in sending the orders or wrong reception information provided by customers).

From the costs mentioned in Table VI, marketers' commission and postal charges are calculated as percentages of monthly sales rate. The other costs such as monthly marketers' salary, click/banner advertisements and returned postal costs are considered exactly as their values in the calibration period. Thus, using equation (11), the monthly return provided by each customer segment during the holdout period can be calculated. The results are shown in Figure 7. It is worth noting again that the fixed costs are not incorporated in calculating these return streams; hence, these values are estimated

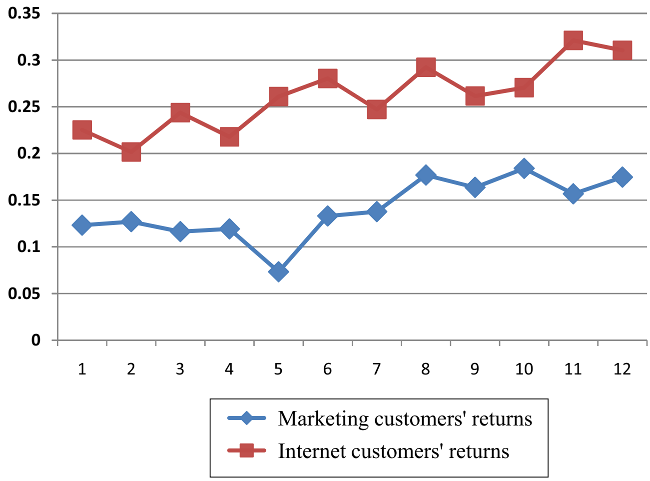


Figure 7.
Monthly returns of two customer segments

higher than their corresponding values in the presence of fixed costs. The comparison of these predicted values versus the realized ones that occurred in the holdout period is illustrated in Table VII.

4.4 Phase 4: estimating the portfolio's possible weights according to the managerial perspective

As mentioned in Section 2, due to some practical restrictions, the proposed portfolio composition may not be achievable for marketing managers. During this research, we obtained the judgements of three managers of our research practice firm about the possible area of customer segments' weights. Three managers were asked to express the maximum achievable segments' share in the next year as a percentage from 0 to 1. These judgements are listed in Table VIII.

Table VII.
Simulated and actual amounts of average monthly returns

Customer segment	Simulated average monthly return	Actual average monthly return
Marketing customers	0.1405	0.1193
Internet customers	0.2610	0.3247

Table VIII.
Decision makers' judgements about the maximum achievable segments' share

Decision maker	Maximum share achievable for internet customer segments	Maximum share achievable for marketing customer segments
DM_1	0.5	0.95
DM_2	0.4	0.7
DM_3	0.65	0.85
The geometric mean of decision makers' judgements	0.5065	0.8268

We used the geometric mean to summarize the views of decision-makers. Assume a_i^k as the view of k^{th} decision-maker about the i^{th} criterion. The geometric mean of all decision-makers is calculated using equation $\bar{a}_i = \Pi_{k=1}^{n} a_i^{(k)1/n}$, where n is the number of decision-makers. Thus, the weights associated with maximum achievable share for the internet and marketing segments would be 0.5065 and 0.8268, respectively. These two values are incorporated into the customer portfolio optimization problem as the upper-level limits for the portfolios' weights.

4.5 Phase 5: identifying the customer portfolio frontier and building the optimal portfolio

Now we can build an efficient frontier by minimizing the return stream volatility while maintaining a certain level of return. For this purpose, the monthly predicted returns in the holdout period are used to compute the efficient frontier and the optimal customer mix. Now, we need to identify the optimal weights of two clusters as $W^* = [w_1^*, \quad w_2^*]$ in such a way that it minimizes the portfolio variance, and its multiplication with the expected returns vector yields a given level of return.

For this purpose, we used the classic M-V model. In addition to equations (8) and (9), we applied a constraint about the upper-level limits for the segments' weights according to the results gained from Phase 4 as $w_i \leq u_i$. To develop the optimization problem and identify the efficient frontier, we used the portfolio object in Matlab financial toolbox to solve this quadratic problem. Our estimation yielded the optimum customer mix in $w_A = 0.36$ and $w_B = 0.64$. The result of this quadratic optimization is illustrated in Figure 8:

The dotted parts in Figure 8 represent the unfeasible area of weight combinations according to the upper-level limit constraint. In this case, the upper limit constraint does not affect on the optimum point. Note that if there were more than two assets in the portfolio, the set of minimum variance portfolio would be as a hyperbolic curve in the return-standard deviation diagram. As shown in Figure 8, the current portfolio

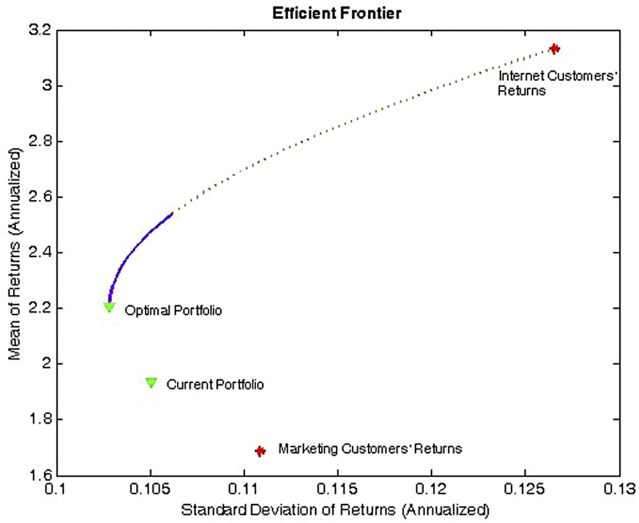


Figure 8.
Efficient frontier with two customer segments

combination provides less monthly returns and more total risk compared with the optimal portfolio. In the optimum point, there is 0.36 share of Segment 1 (internet customers) in the whole portfolio. The results suggest the marketing managers to expand their internet sale and internet customer acquisition efforts in future. Promoting the internet sale about twice more than its current share leads to a growth of about 27 per cent in the total annual returns (Table IX).

Following provides a comparison between the optimal portfolio and the current portfolio in the holdout period. In this comparison, the optimal portfolio performance (extracted from simulated returns) is compared to the actual realized portfolio's performance. Using monthly return and monthly standard deviation of the segments' return as a proxy of risk, we calculated the reward-to-risk ratio in different months of the holdout period. The result is illustrated in Figure 9.

5. Summary and concluding remarks

CPM, taking advantage of different customer profitability trends, tries to find the ideal customer mix with the highest and the most stable return streams. The CPM approach is best suited in the cases that have apparent differences in cash flow volatility trends. It seems that this approach is less appropriate if the trend of return streams generated by different customer segments is highly correlated. Therefore, providing accurate predictions about the returns generated by different customer segments is of great importance to obtain reliable decisions from customer portfolio analysis.

Table IX.
Optimal portfolio performance versus the current portfolio

The portfolio	Customer segment weight Segment 1	Segment 2	Portfolio average return (in Rial 10,000)	Portfolio average standard deviation (in Rial 10,000)
Optimal portfolio	0.36	0.64	0.1839	0.0284
Current portfolio	0.17	0.83	0.1610	0.0290

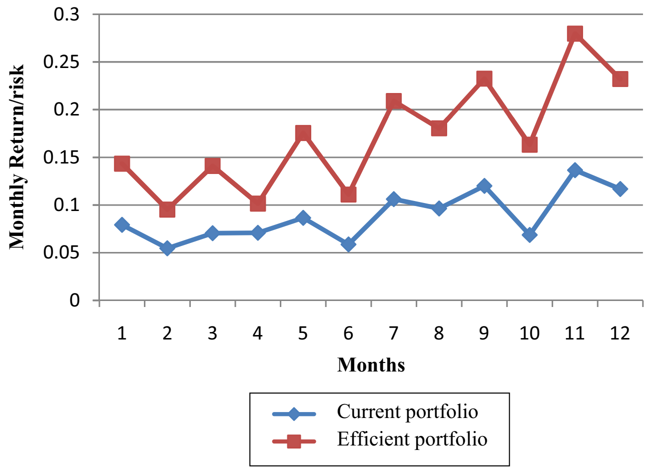


Figure 9.
Testing the optimal portfolio performance versus the actual portfolio

The objective of this research is to extend the customer portfolio research conceptually and empirically by developing a hybrid model that uses stochastic CLV modelling to predict future return streams made by customers and incorporating this prediction in ex ante customer portfolio optimization instead of relying on historical returns. This study examined the portfolio analysis in a cosmetic retailing firm. The two customer segments in this firm had many apparent differences in terms of the type of relationships and their associated costs. Practical results confirm that these two segments have almost different return trends (correlation coefficient = 0.54); hence, it seems that this research practice has the potential to take advantage of diversification. In this study, we developed a novel approach to predict the future returns generated by the customers. For this purpose, using a statistical CLV prediction model, we extracted some statistics describing the customers' purchase behaviour and used them to simulate their future buying behaviour. Applying the M-V optimization on the return streams provided by simulation suggested the optimum mix as 36 per cent of the internet segment and 64 per cent of the marketing segment. Compared with the current portfolio mix (17 per cent of the internet segment and 83 per cent of the marketing segment), the efficient portfolio had much better performance than the current portfolio. In addition, to establish more reliable decisions about the portfolio composition, we only considered the cost terms that are influenced by managerial decisions about customer mix (variable costs).

Also the stability structure of portfolio over time should be noticed. The assets comprising a financial portfolio have a specific identity and are not converted into each other over time. However, if value segmentation of customers is used to identify portfolio components, we cannot ensure that these assets do not convert to each other over time. In such an approach, extending the current segmentation scheme into future time periods may not be valid (The current structure of portfolio could vary over time). This reveals the importance of dynamic portfolio modelling of customer assets, which we noticed as an area for future research. Anyway, this research studied customer portfolio in static form, and we should consider customer segments in a way that ensure they are not convertible to each other over time. Therefore, we consider our two types of customers (retail stores and consumers) as two customer segments of our research practice.

5.1 Theoretical contributions

Application of modern portfolio theory in customer portfolio context has been criticized in the marketing literature (Selnez, 2011; Billet, 2011 and Sackmann *et al.*, 2010). This research tries to consider some of these restrictions by means of providing three important theoretical contributions which is discussed as following.

We extend Tarasi *et al.*'s (2011) work by empirically demonstrating how our knowledge about customer buying behaviour can be incorporated in a customer portfolio problem. This knowledge is used to predict the future performance of customer portfolio. Therefore, the main theoretical implication of this study is to consider the predicted future behaviour of customers rather than their historical behaviour in constructing customer portfolio. The results imply the better performance of this approach versus the historical one. Previous researches in this area used ex post estimation of portfolio returns to extract the efficient frontier (Tarasi *et al.*, 2011; Buhl and Heinrich, 2008; Wangenheim and Lentz, 2005 and Homburg *et al.*, 2009).

Selnez (2011) argued that incorporating customer past purchase behaviour in customer portfolio theory advances this research stream greatly. Responding to his call, we develop a stochastic CLV modelling approach combined with customer portfolio modelling and demonstrate conceptually and empirically how customer purchase pattern could be exploited in this problem. This study also provides a precise measure to predict the return stream associated with different customer segments. We use the return-on-sale ratio to measure this metric. In this ratio, the expected sale of every customer segment is measured using stochastic CLV modelling. Therefore, our second implication is to use the stochastic Pareto/NBD model to extract customer purchase pattern rather than simply assuming a normally distribution of returns. This enables managers to consider the purchase behaviour of different customer segments in portfolio construction. The authors are not aware of any previous research that studies the variability of customer portfolio with non-normal assumption of returns.

Selnez (2011) also argued that in customer portfolio context, the feasible area of weighting schemes have some practical limitations. Our third theoretical contribution is using the managerial judgement to explore the feasible area of portfolio weights. This could improve the mathematical model and its adaption with actual situation.

5.2 Managerial contributions

The results of this study have important implications for marketing managers which are discussed as follows. Marketing managers are always under pressure to represent the return associated with marketing expenditure in different market segments (Rust *et al.*, 2004). Our results provide insights to marketing managers in making managerial decisions concerning the allocation of marketing resources among customer segments.

In common marketing resource allocation perspective, customers with higher profitability are considered more valuable and gain more priority in marketing resource allocation. However, this viewpoint neglects the investment costs and also the interaction of segments on each other within a portfolio. The present study considers different costs associated with different customer segments to extract the return on sale and explore the optimum customer mix.

As it was stated earlier, both magnitude and stability of return streams should be considered simultaneously in analyzing a customer portfolio. If the returns of market segments are predicted properly, the marketing managers will be able to take appropriate decisions on marketing budget allocation and better account for their decisions as well. The results indicate that the predicted return stream provides better performance than historical one. This finding reveals the importance of proper prediction of returns on marketing investment decisions. This enables marketing managers to benefit from using this predictive approach in making investments decisions.

This study also provides an approach to compare the performance of different market segments and their overall performance as a whole portfolio. It also provides a reliable basis to predict the performance of customer portfolio under different resource allocation scenarios. Firms can benefit from customer diversification through first identifying customer purchase patterns and then trying to combine complimentary purchase patterns in a way that yield the greatest and most stable cash flow. We present a practical guide for marketing managers in managing a portfolio consisting of different segments of customers. The implementation of this approach is also very straightforward. Return-on-sale ratio of different market segments is an accessible data for marketing managers. Magnitude and volatility of this metric reveals the purchase pattern of market segments. Therefore, the marketing manager can elect to provide resources to encourage or discourage relationships with different customer segments.

In addition, this research is based on finding the optimum customer mix in a way that provides the most stable cash flow over time. It should be mentioned that other efficient portfolios with different level of risk could be noticed according to the needs of marketing managers.

5.3 Research limitations and future research opportunities

Despite these merits, the study also has some limitations. As discussed earlier, the Pareto/NBD model only considers three pieces of information: "number of transactions", "time interval between the first and last transaction" and "the time interval between the first transaction until now". In other words, this model does not consider historical inter-purchase times as an input parameter. Therefore, it would be valuable if future research could deploy a model that considers the interval between purchases to predict variability.

Another aspect regarding the application of CPM is that common CPM models are designed at one point in time which provides limited view of customer portfolio, whereas CPM is an ongoing, continuous long-term process (Terho, 2009). One proposed approach to respond this research need, is to apply dynamic portfolio modelling instead of static approach where the changes occurred in customer behaviour such as attraction and churn rates or the correlation between their trend could be revised in every time period. Such a model could maintain the flexibility needed in modelling customer purchase patterns and be more applicable to respond to marketing manager needs. Also indicating the drivers of these variations over time is an attractive research area. Few researches such as Wangenheim and Lentz (2005) and recently Tarasi *et al.* (2013) studied about customers' cash flow variability drivers.

In addition, there are some restrictive assumptions in traditional MPT other than aspects discussed in Section 2 that limit the applicability of this approach in CPM. One of them is neglecting from taxes and transactional fees in changing the portfolio combination. It is obvious that in customer portfolio, any adjustment in customer portfolio mix needs to pay for some retention/attraction activities on the target segments, which may significantly differ for different customer segments. Furthermore, it is important to note that if the cost associated with obtaining an extra customer and his/her expected return on investment are different from that of the existing customers within the same segment, one underlying assumption of MPT – the independence of risk/return from customer weights – is violated. In other words, here, in contrast with financial applications, using different allocation scenarios leads to different rate of returns for the assets within a portfolio. The present research provides some steps forward by identifying the optimal portfolio based on customers' current return. But further research is needed to investigate the optimal customer mix while considering non-linear behaviour of customers' return. Here, in line with Tarasi *et al.* (2011), we examined the optimal portfolio as an "ideal customer base" that marketing managers can evaluate its accessibility, associated costs and the resource allocation scenario needed to achieve such a customer mix. However, working on transactional costs associated with adjustments in customer portfolio and incorporating it in portfolio optimization modelling seem to be an open area for further research.

Note

1. For further study about this model and verifying its assumptions, please refer to Schmittlein *et al.* (1987) and Fader *et al.* (2005).

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